ENTROPY LAYER IN THE PROBLEM OF HYPERSONIC FLOW OVER THIN BLUNT BODIES WHICH ARE CLOSE TO TWO-DIMENSIONAL

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In the problem of the hypersonic flow of a nonviscous thermally nonconducting gas over thin blunt bodies which are close to two-dimensional the solution is constructed in the entropy layer. The construction is achieved by a generalization of the method developed in [1] in application to bodies close to two-dimensional. The use of an approximate model identifying the effect of the blunting on the gas with the effect of a concentrated force distributed over the edge is important in the construction. The solution is represented in the form of asymptotic expansions. The equations of the hypersonic theory of small perturbations, which is the null approximation in the process of construction of the solution in the form of a series in powers of a small parameter determined as the square of the relative thickness of the body or the relative width of the perturbed region, are obtained in the null approximation in this case. The surface of the blunt body proves to be singular for the null approximation, since the entropy function $p/\rho^{\varkappa}$ grows without limit as the surface is approached. The attempt to construct the succeeding approximations leads to strengthening of the singularity. This necessitates the use of the method of deformed coordinates (the PLG method). Basic to the latter is the removal of the singularity, which is not inherent to the exact solution of the problem, through asymptotic expansions with respect to a small parameter not only of the unknown variables, but also of the independent variables, with the subsequent determination of the deformation of the independent variables on the basis of the "quenching" of the singularity. Use of the PLG method allows one to construct a solution which is uniformly applicable in the entire stream, including the entropy layer. In practice, the construction of such a solution leads to the determination of the displacement of the streamlines near the surface of the body, as a result of which the singularity is "absorbed" by the body and the solution outside the body proves to be freed of the singularity. In the null approximation this displacement of the streamlines can be determined in closed form.

Suppose a uniform hypersonic stream flows over a thin semi-infinite body blunted along the edge (at the nose).

With respect to the shape of the body (Fig. 1) it is assumed that $\cos(\mathbf{n}, \mathbf{i}) \approx \tau_0$, $\cos(\mathbf{n}, \mathbf{j}) \approx \tau_0^{-1-\nu}$, and $\cos(\mathbf{n}, \mathbf{k}) \approx 1$ except for a small vicinity of the blunt edge (nose): $\tau_0 \ll 1$ is the small parameter and $\nu = 0.1$.

Here $\mathbf{n}$ is the normal to the surface of the body; Lx_i, $i = 1, 2, 3$, is the rectangular Cartesian coordinate system (L is the characteristic length); $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors of the x_1, x_2, and x_3 axes, respectively.

By introducing by analogy with [1] the small parameter $\tau = d^{(1+\nu)/(3+\nu)}$ [d is the characteristic thickness (diameter) of the blunting] and limiting ourselves to the case of $\tau \lessdot \tau_0$ we can examine the asymptotic behavior of the solution in the vicinity of the surface of the body as $\tau \to 0$ on the basis of the perturbation method. In this case we assume that the condition $K = M_\infty \tau \gg 1$ is satisfied.

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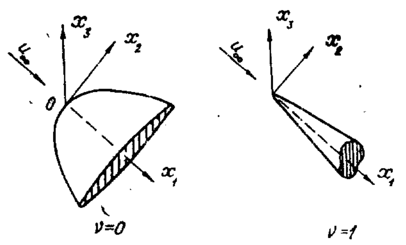

Fig. 1

The system of equations of gasdynamics in the variables "pressure–two stream functions" has the form [2]

$$\frac{\partial u_i}{\partial p} = -\frac{\partial (x_{i+1}, x_{i+2})}{\partial (\varphi, \psi)}, \quad \frac{u_1^2+u_2^2+u_3^2}{2} + \frac{\varkappa}{\varkappa-1}\frac{p}{\rho} = \frac{1}{2} + \frac{1}{\varkappa-1}\mathrm{M}_\infty^{-2}; \tag{1}$$
$$\frac{\partial x_1}{\partial p} u_2 = \frac{\partial x_2}{\partial p} u_1, \quad \frac{\partial x_1}{\partial p} u_3 = \frac{\partial x_3}{\partial p} u_1, \quad \frac{\partial}{\partial p}\left(\frac{p}{\rho^\varkappa}\right) = 0.$$

Here and below an index $i > 3$ should be understood as $i-3$; $\rho_\infty u_\infty^2 p$ is the pressure; $\rho_\infty \rho$ is the density; $u_\infty u_i$ are the components of the velocity vector along the x_i axes, respectively; $\sqrt{\rho_\infty u_\infty}\, L\varphi$, $\sqrt{\rho_\infty u_\infty}\, L\psi$ are the stream functions; $\varkappa$ is the ratio of specific heat capacities of the gas. The vector of the velocity u_∞ of the undisturbed stream is directed along the x_1 axis.

The boundary conditions at the shock wave have the form

$$(\mathbf{i}-\mathbf{V})(\mathbf{in}) = \left(p - \frac{1}{\varkappa}\mathrm{M}_\infty^{-2}\right)\mathbf{n},$$
$$p = \frac{2}{\varkappa+1}(\mathbf{in})^2 + \frac{1-\varkappa}{\varkappa(\varkappa+1)}\mathrm{M}_\infty^{-2}; \tag{2}$$
$$\frac{1}{\rho} = \frac{\varkappa-1}{\varkappa+1} + \frac{2}{\varkappa+1}\mathrm{M}_\infty^{-2}(\mathbf{in})^{-2},$$

where $\mathbf{V}$ is the velocity vector; $\mathbf{n}$ is the unit vector of the normal to the surface of the shock wave.

The solution everywhere in the perturbed region is represented in the form

$$\begin{gathered} x_1 = y_1 + \tau^2 z_1 + \ldots, \quad x_2 = \tau^\nu (y_2 + \tau^2 z_2 + \ldots), \\ x_3 = \tau(y_3 + \tau^2 z_3 + \ldots), \quad \rho = \rho_0 + \tau^2\rho_1 + \ldots, \quad \varphi = \eta, \\ u_1 = 1 + \tau^2 v_1 + \tau^4 w_1 \ldots; \quad u_2 = \tau^{2-\nu}(v_2 + \tau^2 w_2 + \ldots); \\ u_3 = \tau(v_3 + \tau^2 w_3 + \ldots); \quad p = \tau^2(\xi + \tau^2 p_1 + \ldots); \\ \psi = \tau^{1+\nu}(\zeta + \tau^2\psi_1 + \ldots). \end{gathered} \tag{3}$$

The expansion with respect to τ, introduced in [3], for example, is performed in accordance with the orders of the unknown values outside the entropy region.

By converting to the new independent variables ξ, η, ζ in system (1) we obtain the following:

null approximation

$$\frac{\partial v_i}{\partial \xi} = -\frac{\partial (y_{i+1}, y_{i+2})}{\partial (\eta, \zeta)}; \quad v_1 + \frac{\nu v_2^2 + v_3^2}{2} + \frac{\varkappa}{\varkappa-1}\frac{\xi}{\rho_0} - \frac{1}{\varkappa-1}K^{-2} = 0; \tag{4}$$
$$\nu\frac{\partial y_1}{\partial \xi} v_2 = \frac{\partial y_2}{\partial \xi}; \quad \frac{\partial y_1}{\partial \xi} v_3 = \frac{\partial y_3}{\partial \xi}; \quad \frac{\partial}{\partial \xi}\left(\frac{\xi}{\rho_0^\varkappa}\right) = 0;$$

first approximation,

$$\frac{\partial w_i}{\partial \xi} - \frac{\partial p_1}{\partial \xi}\frac{\partial v_i}{\partial \xi} + \frac{\partial (\psi_1, v_i)}{\partial (\zeta, \xi)} = \frac{\partial p_1}{\partial \zeta}\frac{\partial (y_{i+1}, y_{i+2})}{\partial (\eta, \xi)} + \frac{\partial p_1}{\partial \eta}\frac{\partial (y_{i+1}, y_{i+2})}{\partial (\xi, \zeta)} - \frac{\partial (z_{i+1}, y_{i+2})}{\partial (\eta, \zeta)} - \frac{\partial (y_{i+1}, z_{i+2})}{\partial (\eta, \zeta)}; \tag{5}$$
$$w_1 + \nu v_2 w_2 + v_3 w_3 + \frac{v_1^2 + (1-\nu)v_2^2}{2} - \frac{\varkappa}{\varkappa-1}\left(\frac{\xi\rho_1}{\rho_0^2} - \frac{p_1}{\rho_0}\right) = 0;$$
$$w_3\frac{\partial y_1}{\partial \xi} - v_3\left(\frac{\partial p_1}{\partial \xi}\frac{\partial y_1}{\partial \xi} + \frac{\partial \psi_1}{\partial \xi}\frac{\partial y_1}{\partial \zeta}\right) + v_3\frac{\partial z_1}{\partial \xi} + \frac{\partial p_1}{\partial \xi}\frac{\partial y_3}{\partial \xi} + \frac{\partial \psi_1}{\partial \xi}\frac{\partial y_3}{\partial \zeta} = v_1\frac{\partial y_3}{\partial \xi} + \frac{\partial z_3}{\partial \xi};$$
$$\nu\left(w_2\frac{\partial y_1}{\partial \xi} - v_2\left(\frac{\partial p_1}{\partial \xi}\frac{\partial y_1}{\partial \xi} + \frac{\partial \psi_1}{\partial \xi}\frac{\partial y_1}{\partial \zeta}\right) + v_2\frac{\partial z_1}{\partial \xi}\right) + \frac{\partial p_1}{\partial \xi}\frac{\partial y_2}{\partial \xi} \quad \frac{\partial \psi_1}{\partial \xi}\frac{\partial y_2}{\partial \zeta} + (1-\nu)v_2\frac{\partial y_1}{\partial \xi} = v_1\frac{\partial y_2}{\partial \xi} + \frac{\partial z_2}{\partial \xi};$$
$$-\frac{p_1}{\xi} + \varkappa\frac{\rho_1}{\rho_0} + \frac{\rho_0^\varkappa}{\xi}\frac{\partial}{\partial \zeta}\left(\frac{\xi}{\rho_0^\varkappa}\right)\psi_1 = F_0(\eta, \zeta).$$

The expansions of the conditions (2) are obvious and are not presented here. The boundary condition at the surface of the body in the null approximation has the form

$$y_3=f(y_1, y_2) \quad \text{at} \quad \zeta=0.$$

Another initial condition must be added to the system of equations of the null approximation [1]. By analogy with [1] we assign the initial condition determining the effect of the blunting on the gas as a concentrated force distributed over the edge without allowance for the dimensions of the blunting. In the case of $\nu = 1$ we are limited to the consideration of bodies with axisymmetric blunting washed by a jet flow which crosses the front in the vicinity of the blunting. The asymptotic representation of the shape of the front in the vicinity of the blunting is known [4] in this case:

$$y_1 = \Phi(\eta, 1)\zeta; \quad y_2^2 + y_3^2 = 2\zeta; \quad y_2 = \mathrm{tg}(\eta) y_3.$$

In the case of $\nu = 0$ we are limited to bodies having a leading edge with a shape $\{x_1 = y_0(\eta), x_2 = \eta, x_3 = \tau Z(\eta)\}$ which has a small enough curvature as $\tau \to 0$ that, because of the nature of the blunting, the density distribution of the concentrated force over the edge is a smooth function. The asymptotic expression for the shape of the front in the vicinity of the blunting for the upper part of the stream ($\zeta \geq 0$) is given in the form

$$y_1=y_0(\eta)+\Phi(\eta, 0)\zeta^{3/2}; \; y_2=\eta; \; y_3=Z(\eta)+\zeta.$$

It is analogous for the lower part ($\zeta \leq 0$). With the indicated limitations the shape of this front can be obtained by the method of local sweepback [5].

Note. In the general case where the surface of the shock wave in the vicinity of the blunting has the form

$$y_1= \Phi(\eta, 1)\zeta; \quad y_2^2 + y_3^2 = 2\zeta^{\alpha};$$
$$y_2=\mathrm{tg}(\eta)y_3; \; \nu=1; \; y_1=y_0(\eta)+\Phi(\eta, 0)\zeta^{\beta};$$
$$y_2=\eta; \; y_3 = Z(\eta)+\zeta; \; \nu = 0,$$

the expansions (3) are inapplicable for the study of the behavior of the solution in the entropy layer if $0 < \alpha < 1$ and $0 < \beta^{-1} < 2/3$, since no solution of the null approximation exists for these values of the parameters α and β [6]. If $1 < \alpha < 2$ and $2/3 < \beta^{-1} < 1$ then the expansions (3) constructed by the system developed below are not asymptotic with respect to the parameter τ in the entropy layer. In the approximate formulation of the problem, however, the shape of the shock wave in the vicinity of the blunting is universal $\alpha = 1$, $\beta = 2/3$, and the surface of the body in the null approximation is sharpened along the edge (the blunting thickness is neglected). As will be shown below, in this case the expansions (3) are asymptotic with respect to the parameter τ in the entropy layer.

Using the arbitrariness in the system of equations of the first approximation, we will assume that the function p_1 is sufficiently smooth with respect to ζ and we will determine it at the front in such a way that the function ξ_* in the boundary condition $p_* = \tau^2(\xi + \tau^2 p_1 + \ldots)_*$ has the form $\xi_* = \xi_0(\eta, \zeta)$, i.e., does not depend on τ. As for the function ψ_1, we will set $\psi_1(\xi_0\,\eta, \zeta) = 0$, thereby excluding deformation by the surface of the flow at the front.

Since the shape of the front is known,

$$\frac{1}{\rho_0} = \frac{\varkappa-1}{\varkappa+1}\left(\frac{1}{\varkappa+1}\Phi^{-2}(\eta, \nu)\right)^{1/\varkappa}\left(\frac{8}{9}\right)^{(1-\nu)/\varkappa}\xi^{-1/\varkappa}\zeta^{-1/\varkappa} + \ldots \approx - A(\eta, \nu)\,\xi^{-1/\varkappa}\zeta^{-1/\varkappa} \quad \text{as} \quad \zeta \to 0. \tag{6}$$

By representing the unknown solutions in the form of asymptotic expansions with respect to the system of functions $F_{ml}(\zeta)$, where $F_{mj}F_{ml}^{-1} \to 0$ as $\zeta \to 0$, $j > l$, $(m = 1, \ldots, 7; \; l = l_{0m}, \ldots, \infty)$, we obtain:

a) $\nu = 0$, null approximation

$$y_1 = y_{10}(\xi, \eta) + \frac{\partial^2 f}{\partial \alpha^2}\frac{\partial y_{10}}{\partial \xi}\zeta + \ldots, \tag{7}$$

where y_{10} is an arbitrary function;

$$y_2 = \eta, \quad y_3 = f - \frac{\varkappa}{\varkappa - 1} A\xi^{-1/\varkappa}\zeta^{(\varkappa-1)/\varkappa} + \ldots;$$
$$v_1 = \frac{\varkappa}{\varkappa - 1} A\xi^{(\varkappa-1)/\varkappa}\zeta^{-1/\varkappa} + \ldots;$$
$$v_3 = \frac{\partial f}{\partial \alpha} + \frac{1}{\varkappa - 1} A\left(\frac{\partial y_{10}}{\partial \xi}\right)^{-1}\xi^{-1-1/\varkappa}\zeta^{(\varkappa-1)/\varkappa} + \ldots.$$

The function $f(\alpha, \beta)$ determined above and its derivatives are taken with the values $\alpha = y_{10}$, $\beta = \eta$. The expression for the velocity v_2 has the form

$$v_2 = -\int\left(\frac{\partial y_1}{\partial \eta}\frac{\partial v_1}{\partial \xi} + \frac{\partial y_3}{\partial \eta}\frac{\partial v_3}{\partial \xi}\right)d\xi + \gamma(\eta, \zeta).$$

To determine the behavior of the function γ as $\zeta \to 0$ we introduce in the latter equality $\xi \to \xi_0(\eta, \zeta)$. Using the boundary conditions at the front and the condition $y_1(\infty, \eta, 0) = y_0(\eta)$, we obtain $\gamma \to 0$ as $\zeta \to 0$. Then we can write the expansion of the function as

$$v_2 = -A\int \frac{\partial y_{10}}{\partial \eta}\xi^{-1/\varkappa}d\xi\zeta^{-1/\varkappa} + \ldots \tag{8}$$

b) $\nu = 1$, null approximation. From (4) it follows identically that

$$\frac{\partial y_1}{\partial \eta}\frac{\partial v_1}{\partial \xi} + \frac{\partial y_2}{\partial \eta}\frac{\partial v_2}{\partial \xi} + \frac{\partial y_3}{\partial \eta}\frac{\partial v_3}{\partial \xi} \equiv 0.$$

Seeking only functions y_i which are bounded for small ζ in accordance with the boundary conditions and allowing for their identical boundedness as $\zeta \to 0$ (except for $\partial v_1/\partial \xi$), we obtain $\partial y_1/\partial \eta = 0$ at $\zeta = 0$. This means that if the body is washed by a jet flow which crosses the wave front in the vicinity of the blunting, then the pressure at the body in the null approximation depends only on the coordinate y_1. The latter is a reflection of the fact that in the corresponding two-dimensional nonsteady motion of a gas from a "powerful explosion" the pressure gradient at the surface of the body is equal to zero at every moment $t > 0$ if the gas density at the body is equal to zero.

With allowance for this, we have

$$\begin{gathered} y_1 = y_{10}(\xi) + y_{11}\zeta + \ldots; \\ v_1 = \frac{\varkappa}{\varkappa - 1} A\xi^{(\varkappa-1)/\varkappa}\zeta^{-1/\varkappa} + \ldots; \\ y_j = y_{j0} + y_{j1}\, g(\zeta) + \ldots; \\ \frac{\partial y_{10}}{\partial \xi} v_j = \frac{\partial y_{j0}}{\partial \xi} + \frac{\partial y_{j1}}{\partial \xi} g(\zeta) + \ldots, \quad j = 2, 3. \end{gathered} \tag{9}$$

For the determination of the required functions we have the following system of equations:

$$\begin{gathered} -\frac{\partial f}{\partial \beta}\frac{\partial f}{\partial \xi}\left[\frac{\partial f}{\partial \alpha} + \frac{\partial f}{\partial \beta}\frac{\partial y_{20}}{\partial \xi}\left(\frac{\partial y_{10}}{\partial \xi}\right)^{-1}\right] = \frac{\partial}{\partial \xi}\left[\frac{\partial y_{20}}{\partial \xi}\left(\frac{\partial y_{10}}{\partial \xi}\right)^{-1}\right); \quad y_{30} = f; \\ -\frac{\partial f}{\partial \beta}\frac{\partial y_{20}}{\partial \eta} y_{11} = \frac{\partial}{\partial \xi}\left[\frac{\partial y_{20}}{\partial \xi}\left(\frac{\partial y_{10}}{\partial \xi}\right)^{-1}\right], \end{gathered} \tag{10}$$

where $y_{10}(\xi)$ is an arbitrary function and $f(\alpha, \beta)$ is determined above. Both the function f and its derivatives are taken with the values $\alpha = y_{10}$, $\beta = y_{20}$:

$$\begin{gathered} y_{21}\frac{\partial f}{\partial \beta} - y_{31} = \begin{cases} \frac{\varkappa}{\varkappa - 1}\left(\frac{\partial y_{20}}{\partial \eta}\right)^{-1} A\xi^{-1/\varkappa}, & \text{if} \quad g = \zeta^{(\varkappa-1)/\varkappa} \\ 0, y_{21} \neq 0, \ y_{31} \neq 0, & \text{if} \quad \zeta^{(\varkappa-1)/\varkappa} = o(g) \\ & \qquad \text{as} \quad \zeta \to 0; \end{cases} \\ \frac{\partial}{\partial \xi}\left[\frac{\partial y_{21}}{\partial \xi}\left(\frac{\partial y_{10}}{\partial \xi}\right)^{-1}\right] + \frac{\partial f}{\partial \beta}\frac{\partial}{\partial \xi}\left[\frac{\partial y_{31}}{\partial \xi}\left(\frac{\partial y_{10}}{\partial \xi}\right)^{-1}\right] = y_{11}\left[\frac{\partial y_{21}}{\partial \eta}\frac{\partial f}{\partial \beta} - \frac{\partial y_{31}}{\partial \eta}\right] + \frac{\partial y_{11}}{\partial \eta}\left[y_{31} - \frac{\partial f}{\partial \beta} y_{21}\right]c; \\ c = \lim g^{-1}\zeta\frac{dg}{d\zeta} \quad \text{as} \quad \zeta \to 0. \end{gathered} \tag{11}$$

One can show that for axisymmetric bodies $y_{21}(\partial f/\partial\beta) - y_{31} \neq 0$, if $y_{21} \neq 0$ or $y_{31} \neq 0$. Therefore, for bodies which differ little from axisymmetric bodies we can set $g = \xi^{(\varkappa-1)/\varkappa}$.

Following [1], we remove in the first approximation the boundary condition at the surface of the body and set up the problem of determining the flow behind the shock wave obtained in the null approximation.

The expression for the density in the first approximation takes the form

$$-\frac{p_1}{\xi} + \varkappa\frac{\rho_1}{\rho_0} - \frac{1}{\zeta}\psi_1 = \frac{1}{2}\Phi^{-2}(\eta, \nu)\,\zeta^{-1}\left[\frac{8}{9}\left(1 + \left(\frac{dy_0}{d\eta}\right)^2\right)\right]^{1-\nu} = F_0(\eta, \zeta, \nu).$$

Substituting the expressions for w_1 and ρ_1/ρ_0 into the first equation of system (5) and reducing like terms, we obtain the greatest singularity in the coefficients of the first equation in the case of $\psi_1 \equiv 0$. Therefore we can chose the arbitrary function ψ_1 in such a way that the singularity in the coefficients of the first equation is integrable. For this it is enough to set

$$\frac{\partial\psi_1}{\partial\zeta} - \frac{1}{\varkappa}\left(F_0 + \frac{1}{\zeta}\psi_1\right) - \left(\frac{\varkappa}{\varkappa-1}\xi^{(\varkappa-1)/\varkappa} + (1-\nu)\frac{\partial y_{10}}{\partial\eta}\int\frac{\partial y_{10}}{\partial\eta}\xi^{-1/\varkappa}d\xi\right)A\zeta^{-1/\varkappa} = 0.$$

The solution of the latter equation with allowance for the condition $\psi_1 = 0$ at $\xi = \xi_0$ has the form

$$\psi_1 = -\frac{1}{2}\Phi^{-2}(\eta, \nu)\left[\frac{8}{9}\left(1 + \left(\frac{dy_0}{d\eta}\right)^2\right)\right]^{1-\nu} + 0\left(\zeta^{(\varkappa-1)/\varkappa}\right). \tag{12}$$

Determining the displacement of the flow surface $\psi = 0$, we obtain

$$\zeta \approx \tau^2\frac{1}{2}\Phi^{-2}(\eta, \nu)\left[\frac{8}{9}\left(1 + \left(\frac{\partial y_0}{\partial\eta}\right)^2\right)\right]^{1-\nu} = \tau^2\zeta_0(\eta, \nu).$$

Let us examine the behavior of the other functions.

c) $\nu = 0$, first approximation. Using the null approximation, one can show that $\partial y_1/\partial\xi \sim \xi^{-5/2}$ as $\zeta \to 0$ and $\xi \to \xi_0$. From the latter and the boundary condition for z_2 (it is easy to show that $z_2 = 0$ at $\xi = \xi_0$) it follows that the arbitrary function r in the expression $z_2 = \int v_2 \frac{\partial y_1}{\partial\xi}d\xi + r(\eta, \zeta)$ approaches zero as $\zeta \to 0$

$$z_2 = -A\int\left(\frac{\partial y_{10}}{\partial\xi}\int\frac{\partial y_{10}}{\partial\eta}\xi^{-1/\varkappa}d\xi\right)d\xi\zeta^{-1/\varkappa} + \ldots = -Y(\eta, \xi)\,\zeta^{-1/\varkappa} + \ldots. \tag{13}$$

Therefore, at the displaced surface $\zeta = \tau^2\zeta_0$ we have the following distribution of the unknown functions:

$$\begin{gathered}\varphi = \eta;\quad x_1 = y_{10}(\xi, \eta) - \frac{\partial y_{10}}{\partial\eta}Y\zeta_0^{-1/\varkappa}\tau^{2-2/\varkappa} + \ldots;\\ x_2 = \eta - Y\zeta_0^{-1/\varkappa}\tau^{2-2/\varkappa} + \ldots;\\ \frac{x_3}{\tau} = f - \left(\frac{\varkappa}{\varkappa-1}A\xi^{-1/\varkappa}\zeta_0 + \frac{\partial f}{\partial\eta}Y\right)\zeta_0^{-1/\varkappa}\tau^{2-2/\varkappa} + \ldots\end{gathered} \tag{14}$$

Here the terms of order $\tau^{4(\varkappa-1)/\varkappa}$ and higher are not written. The function f is taken with the values $\alpha = y_{10}$, $\beta = \eta$. The concrete form of the other functions, which have the order of $u_1 \sim 1 + 0(\tau^{2-2/\varkappa})$, $u_2 \sim \tau^{2-2/\varkappa}$, $u_3 \sim \tau$, $\rho \sim \tau^{2/\varkappa}$ and $p \sim \tau^2$ at the displaced contour, is not presented.

We note only that with such a choice of the function ψ_1 the expansions of the unknown values for $\psi \geq 0$ preserve the asymptotic properties with respect to the parameter τ. From the form of the function Y it follows that the "three-dimensional" effect in the distribution of the streamlines in the vicinity of the surface of the body is caused by the deviation of both the shape of the edge and the shape of the body from a flat shape.

d) $\nu = 1$, first approximation. The asymptotic representation of the solution has the form

$$z_1 = z_{10} \max(\zeta F,\ g) + \dots;\ w_1 = w_{10} \max(F,\ \zeta^{-2/\varkappa}) + \dots;$$
$$z_j = z_{j0} F + \dots;\ w_j = w_{j0} \max(F,\ \zeta^{-1/\varkappa}) + \dots,\ j = 2.3.$$

If there are proper solutions of the type $g = o(F)$ as $\xi \to 0$ (the function g is determined in the null approximation), then necessarily

$$\frac{\partial y_{20}}{\partial \eta} z_{30} - \frac{\partial y_{30}}{\partial \eta} z_{20} = 0;\quad z_{30} \neq 0;\quad z_{20} \neq 0.$$

The latter equation, as in the case of the null approximation, is not satisfied for bodies which are close to axisymmetric. For them at the displaced surface we have

$$x_1 = y_{10}(\xi) + \dots;\quad x_2 = \tau\left(y_{20} + y_{21}\zeta_0^{(\varkappa-1)/\varkappa}\tau^{2-2/\varkappa} + \dots\right);$$
$$x_3 = \tau\left(f + y_{31}\zeta_0^{(\varkappa-1)/\varkappa}\tau^{2-2/\chi} + \dots\right).$$

Here the terms of order $\tau^{4(\varkappa-1)/\varkappa}$ and higher are not written. The function f is taken with the values $\alpha = y_{10}$, $\beta = y_{20}$. The concrete form of the other functions, which have the order of $u_1 \sim 1 + 0(\tau^{2-2/\varkappa})$, $u_2 \sim u_3 \sim \tau$, $\rho \sim \tau^{2/\varkappa}$ and $p \sim \tau^2$ at the displaced contour, is not presented. The expansions with respect to the parameter τ for $\psi \geq 0$ preserve the asymptotic properties, and the problem of the determination of the displaced contour and the distribution of the unknown values at it, if the function $y_{10}(\xi)$ is known, is reduced to the determination of the solution of the system of ordinary differential equations (10), (11).

Turning to the expansions (3) and the results of the study of the first approximation, we can conclude that the solution of the null approximation for a body with a displaced contour is free from singularities and is uniformly applicable in the entire stream with a relative error on the order of $\tau^{2(\varkappa-1)/\varkappa}$. Thus, the method used allows one to improve the solution at finite distances from the leading edge. In doing this the asymptotic solution is set up in the entropy layer in accordance with Eqs. (6)-(13), if the pressure distribution over the body is known in the null approximation. As follows from (6)-(13), a slight variation in the pressure in the direction orthogonal to the surface of the body is observed in the general case. If the body is close to axisymmetric then the pressure at the deformed surface depends only on the longitudinal coordinate x_1, i.e., the entropy layer does not restrain the pressure drop in the circumferential direction.

In conclusion, let us examine the qualitative pattern of flow over a flat plate with the normal orthogonal to the plane $x_3 = 0$. We set the number M_∞ equal to infinity and we take the shape of the blunting in the form

$$x_2 = \pm(\alpha_1 x_1^\varepsilon + \alpha_2 x_1);\quad x_1 \geqslant 0;$$
$$0 < \varepsilon \leqslant 1/2;\ 0 \leqslant x_3 \leqslant \tau^3;\ \alpha_i \geq 0. \tag{15}$$

The values $1/2 < \varepsilon < 1$ are excluded from consideration, since the condition of boundedness of the curvature of the edge at the apex of the plate is violated in these cases. In the vicinity of the blunting we can represent the shape of the shock wave calculated on the basis of the model with a concentrated force (Newton's equation was used to calculate the latter) using the principle of local sweepback,

$$x_3 = \tau c_0 \left\{\left[1 + \left(\frac{dy_0}{dx_2}\right)^2\right]^{-1/2} (x_1 - y_0)\right\}^{2/3}. \tag{16}$$

Here and below the expressions for the coefficients $c_i > 0$ are not made concrete because of the awkwardness. The function $y_0(x_2)$ is found from (15) with x_1 replaced by y_0. The function y_{10}, which is easily reconstructed from the pressure distribution over the body in the null approximation, has the form

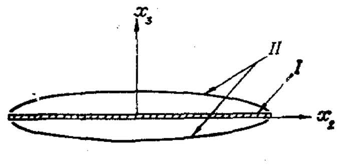

Fig. 2

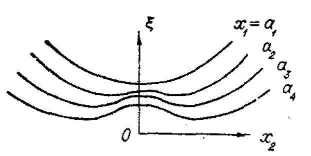

Fig. 3

$$y_{10} = y_0 + c_1\left[1 + \left(\frac{dy_0}{dx_2}\right)^2\right]^{-1} \xi^{-3/2}.$$

Eliminating the function $Y(\xi, \eta)$ from (14), we obtain the equation for the shape of the displaced contour:

$$\frac{x_3}{\tau} = c_2\left(1 + \left(\frac{dy_0}{dx_2}\right)^2\right)^{-1/3\varkappa} (x_1 - y_0)^{2/3\varkappa} \tau^{2-2/\varkappa}. \tag{17}$$

In application to the inverse problem, Eq. (17) describes the nature of the variation in the shape of the surface of the body for a given shape of the shock wave (16). The qualitative nature of the variation in the shape of the displaced contour in the plane x_1 = const is presented in Fig. 2 (I is the cross section of the plate and II is the cross section of the displaced contour). Curve II, symmetrical relative to the coordinate axes, is a monotonically decreasing function for $x_2 > 0$ and $x_3 > 0$ which does not have points of inflection in the case when

$$3\varkappa + (3\varkappa - 4)\left(\frac{dy_0}{dx_2}\right)^2 > 0.$$

The question of the presence of points of inflection in the general case requires a more detailed study of Eq. (17).

Let us write the projections of the streamlines η = const of the displaced contour onto the plane $x_3 = 0$:

$$\begin{cases} \eta = x_2 - f_0^{1+1/\varkappa}\left[c_3 \frac{dy_0}{dx_2} - c_4 \frac{df_0}{dx_2} \xi^{-3/2}\right] \xi^{-1/2-1/\varkappa} \tau^{2-2/\varkappa}; \\ x_1 = y_0 + c_1 f_0 \xi^{-3/2}; \quad f_0 = \left(1 + \left(\frac{dy_0}{dx_2}\right)^2\right)^{-1}. \end{cases}$$

Taking η as a function of the coordinates x_1 and x_2 in them, we obtain $\partial\eta/\partial x_1 < 0$, i.e., the gas flows out from the plane of symmetry $x_2 = 0$. By examining the behavior of the pressure at the displaced contour in the cross sections $x_1 = a_i$ (a_i are constants and $a_i < a_{i+1}$), we can establish the presence of regions of reduced pressure in the vicinity of the plane of symmetry $x_2 = 0$ (Fig. 3), which does not contradict the results of [7, 8]. In the case of $\varepsilon = 1/2$ these regions of reduced pressure originate near the apex, at a distance equal to half the radius of curvature of the edge at the apex of the plate.

LITERATURE CITED

1. V. V. Sychev, "On the method of small perturbations in problems of the flow of a hypersonic stream over thin blunt bodies," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1962).
2. V. G. Dulov, "On the equations of three-dimensional steady gas flows in special dynamic variables," in: Proceedings of Second International Colloquium on the Gasdynamics of Explosions and of Reacting Systems, Novosibirsk, 1969. Section on Numerical Metods in Gasdynamics [in Russian], Vychisl. Tsentr. Akad. Nauk SSSR, Moscow (1969).
3. W. H.-T. Loh, Modern Developments in Gas Dynamics, Plenum Press, New York (1969).
4. G. G. Chernyi, Gas Flows with High Supersonic Velocity [in Russian] Fizmatgiz, Moscow (1959).
5. M. D. Ladyzhenskii, Three-Dimensional Hypersonic Gas Flows [in Russian], Mashinostroenie, Moscow (1968).
6. S. S. Grigoryan, "The Gauchy problem and the piston problem for the equations of one-dimensional nonsteady gas movements (self-similar solutions)," Prikl. Mat. Mekh., 22, No. 2 (1958).
7. M. H. Bertram and A. Henderson, "Recent hypersonic studies of wings and bodies," ARSJ. 31, No. 8, 1129 (1961).
8. V. V. Mikhailov, "Flow over a delta wing having blunt edges with strong compression in the shock layer," Uch. Zap. Tsentr. Aéro-Gidrodinam. Inst., 3, No. 3, (1972).